

# Integrated Modeling of High Speed Passenger and Freight Train Operation Planning on a Shared-track Corridor

Ahmadreza Talebian, Bo Zou  
University of Illinois at Chicago

Presentation at CMAP  
April 2, 2014

# Acknowledgement

This research is partially supported by the  
Illinois Department of Transportation  
through the Urban Transportation Center at  
the University of Illinois at Chicago

# Outline

- Background
- Literature review
- The train schedule model
  - Modeling approach
  - Objective function
  - Constraints
- Solution approach
- Results and discussions
- Summary

# Background

- Train scheduling problem has been investigated for a long time
- The problem is gaining growing attention in the US recently due to the resurgence of passenger rail on passenger freight shared use corridors
- On shared-use tracks, integrated train schedule planning problem needs to address:
  - Scheduling priority between two types of trains
  - Train meets
  - Train overpasses

# Background

- Illinois HSR: Chicago-St. Louis (current phase)
  - **Single track** (with sidings)
  - Shared passenger and freight use
  - High speed passenger trains operating at 110 mph



Source: IDOT (2014)

# Literature review

- Three approaches in train scheduling: analytical, simulation, and optimization

Authors	Objective	Modeling priority	Discrete time	Model structure
Brännlund et al., 1998	Min schedule deviation	Y	Y	ILP
Oliveira and Smith, 2000	Min schedule deviation	N	Y	---
Caprara et al., 2002	Min schedule deviation	N	Y	ILP
Caprara et al., 2006	Min schedule deviation	Y	Y	ILP
Canca et al., 2011	Min passengers' waiting time	N	Y	INLP
Harrod, 2011	Max total utility of trains	Y	Y	ILP
Liu and Kozan, 2011	Min schedule makespan	Y	N	MILP

- Discrete time modeling is dominant
- Most of the studies use an “ideal timetable”

# Literature review

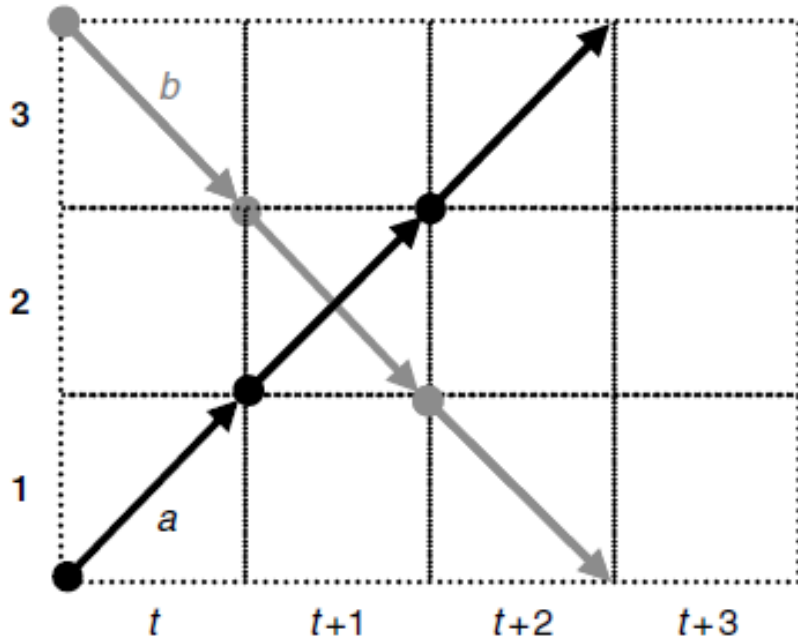
- Question: What is an “ideal schedule”?
- Very limited efforts in obtaining ideal train schedules
- Traveler schedule convenience: an important factor in designing passenger trains schedules
- While schedule delay has been investigated in aviation and transit planning, it is largely absent in passenger rail modeling
- One major contribution of this study: incorporating passenger schedule delay in shared rail corridor analysis

# Modeling approach

- Discretize time and space (line distance)
- Define a set of train paths on a railway line as a multi-commodity flow on a *directed hypergraph*
  - A hypergraph is a graph in which the definition of an edge is expanded to include any non-empty subset of nodes
  - Hypergraph can address path conflicts that conventional discrete time dynamic graphs are unable to address

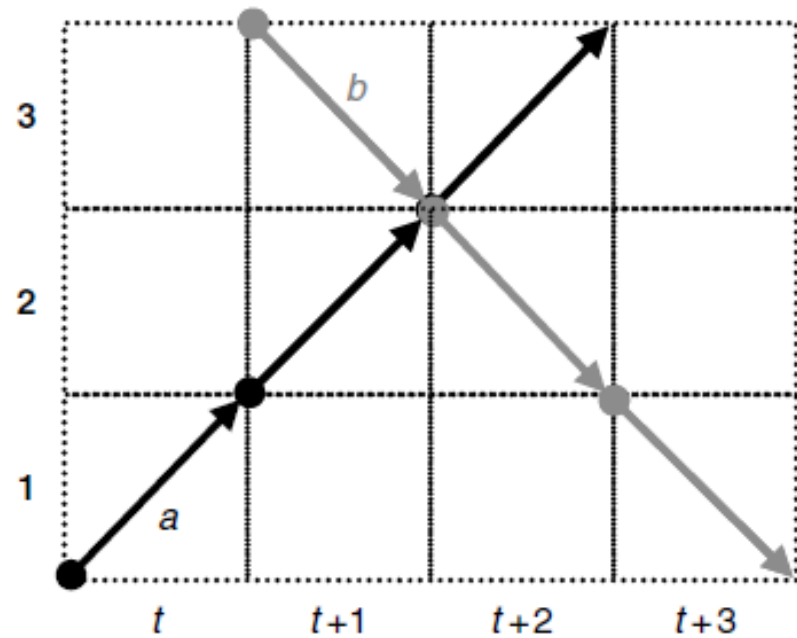


# Deficiency of the traditional block occupancy scheduling model



Intended rule enforcement

What traditional block occupancy scheduling models can deal with:  
**Conflicts within a block**



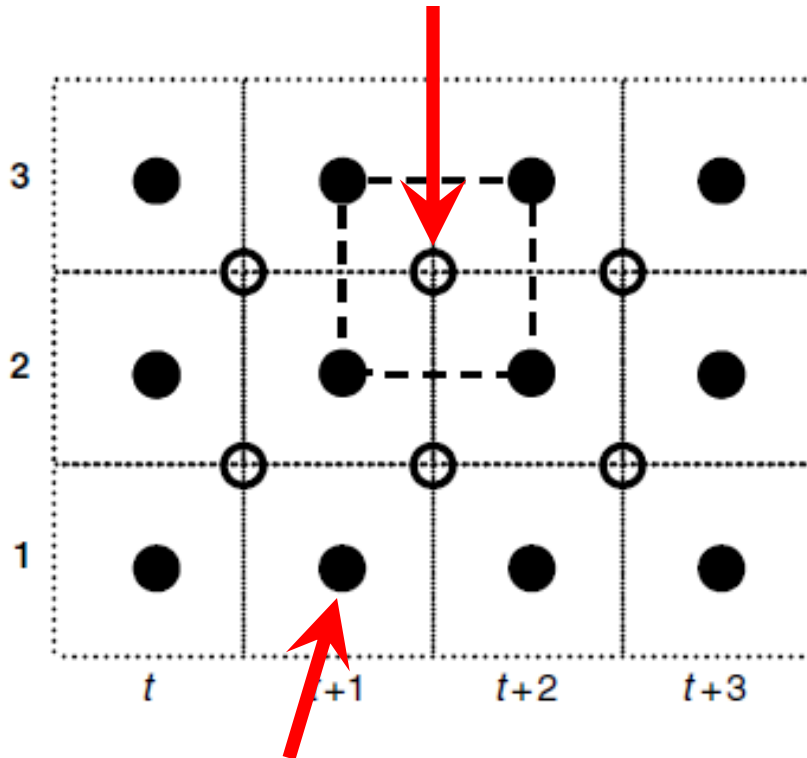
Unintended violation

Source: Harrod (2011)

What traditional block occupancy scheduling models cannot deal with:  
**Conflicts during train transitions between blocks**

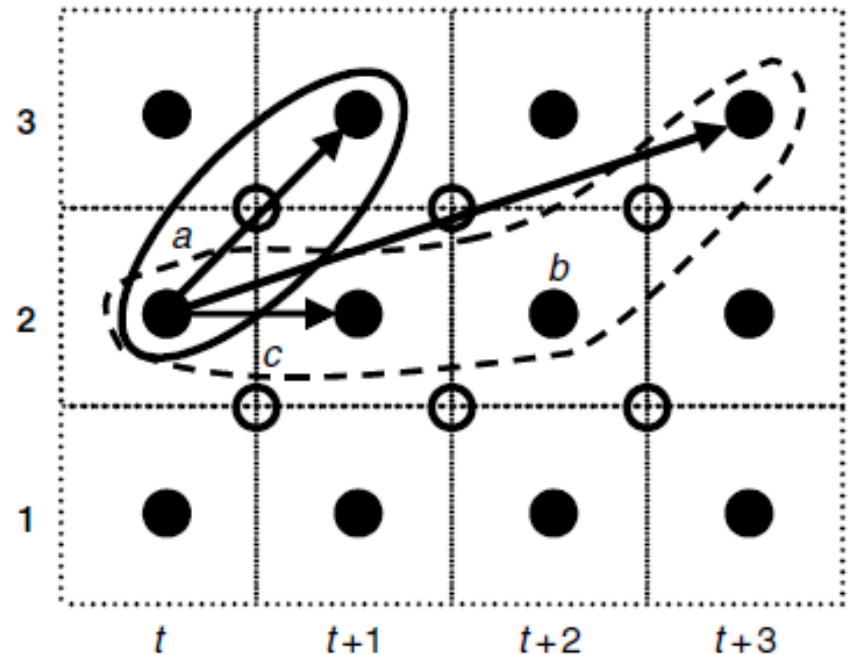
# Using hypergraphs to train schedule modeling

Transition nodes



Block occupancy nodes

Hyperarcs **b** consists of four block occupancy nodes  $\{(2,t), (2,t+1), (2,t+2), (3,t+3)\}$  and a transition node  $(2,t+2)$



Source: Harrod (2011)

Because of explicit enumeration of occupancy and transition nodes, capacity constraints and conflict both within blocks and during transition can be explicitly considered.

# Problem formulation: planning objective

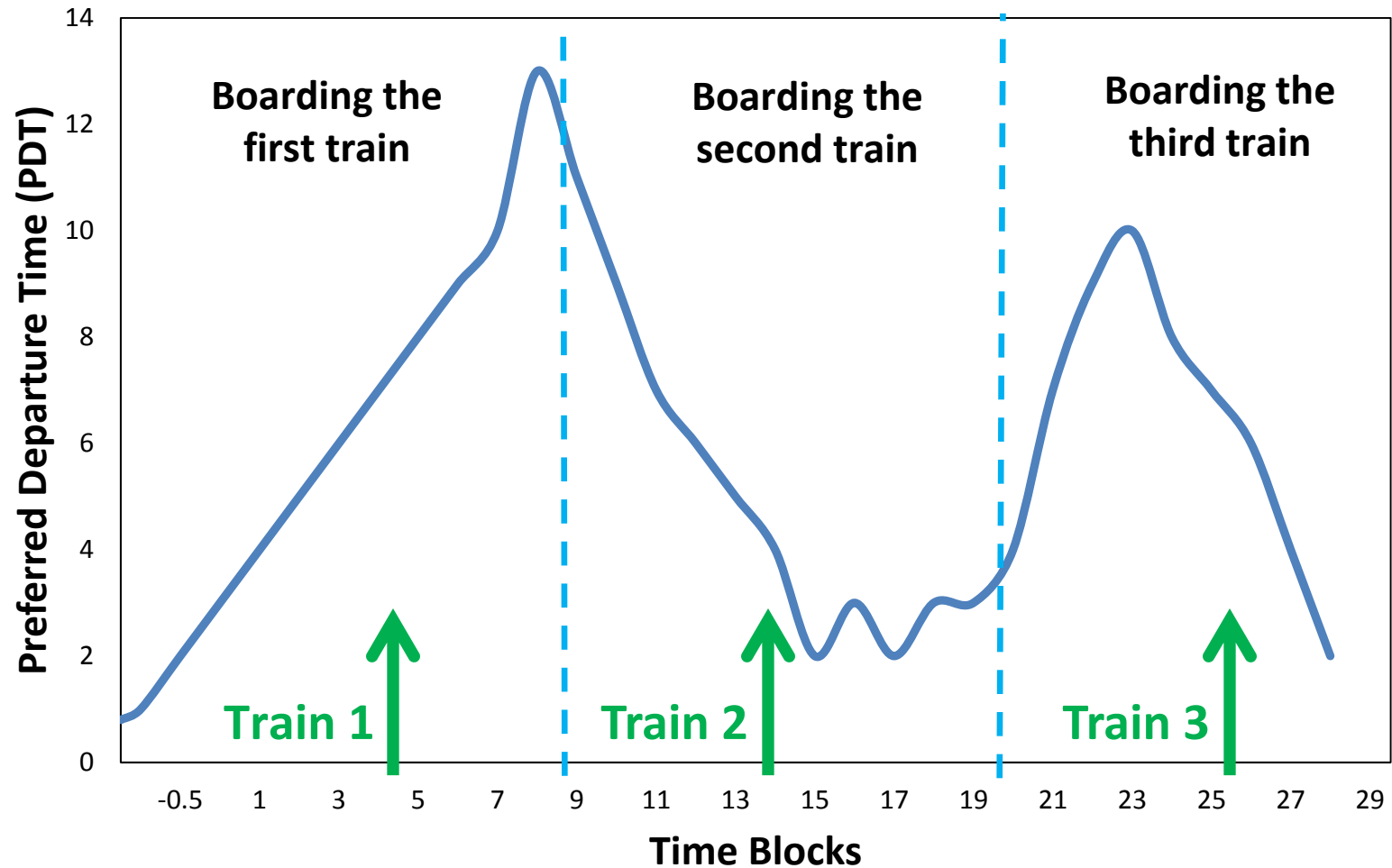
- We approach the train scheduling problem from a central planner's perspective
  - Pax train operating cost
  - Freight train cost
  - Pax in-vehicle travel time
  - Pax schedule delay
- By Public Law 110-432 (110 Congress, 2008), Amtrak trains have priority over all freight trains

# Problem formulation: planning objective

- We approach the train scheduling problem from a central planner's perspective
  - Pax train operating cost
  - Freight train cost
  - Pax in-vehicle travel time
  - **Pax schedule delay**

**Optimal train schedule is a function  
of passenger demand profile**

# Passenger demand profile



# Objective function

- A Two-level optimization approach: assuming that passenger trains have high scheduling priority

**Level 1:**     min (Pax train related cost)

**Level 2:**     min (Freight train related cost)  
                  s.t. Passenger train schedule

# Objective function: Level 2

## Freight train related cost

$$\begin{aligned} \text{Min} \quad & \sum_{r \in R} \sum_{(p_o^r, j, t_i, t_j) \in \Psi^r} \left[ \boxed{(c_e^r (t_i - p_e^r) - c_p^r)} x_{p_o^r, j, t_i, t_j}^r + \sum_{r \in R} \sum_{(i, j, t_i, t_j) \in \Psi^r \mid i=j} \boxed{c_s^r x_{i, j, t_i, t_j}^r} \right] \\ & \text{Departure delay cost} \quad \quad \quad \text{En-route delay cost} \\ & \quad \quad \quad \text{Foregone demand cost} \end{aligned}$$

# Objective function: Level 1

## Passenger train related cost

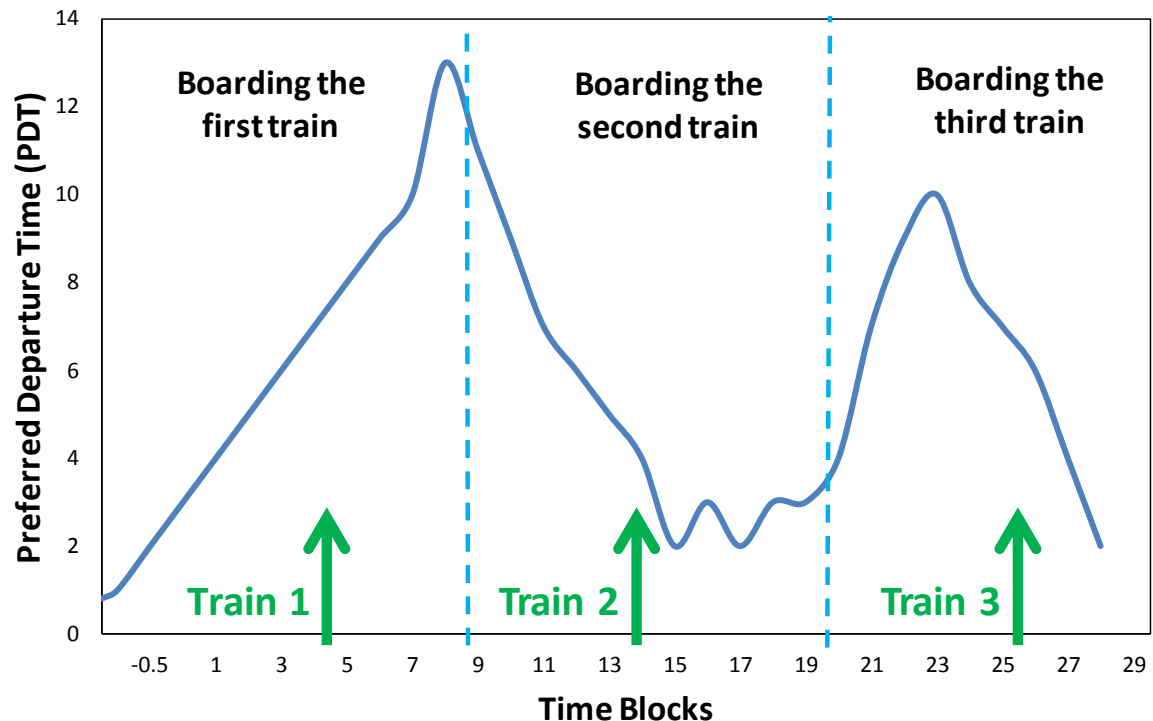
- Because passenger trains have scheduling priority, there is no departure or en-route delay
- Passenger in-vehicle travel time and passenger train operating cost remain constant
- Only need to consider passenger schedule delay cost



# Objective function: Level 1

## Passenger schedule delay cost

- Each O-D pair has a passenger demand profile (Preferred Departure Time)
- Passengers are served by a predetermined number of trains



# Objective function: Level 1

## Passenger train related cost

$$\begin{aligned}
 & \text{Min} \sum_{\substack{pq \in PQ \\ (p_o^{Opq(1)}, j, t_{p_o}^{Opq(1)}, t_j) \in \psi^{Opq(1)}}} c_d^{pq} \left( w_{t_{p_o}^{Opq(1)}, L}^{Opq(1)} + w_{t_{p_o}^{Opq(1)}, R}^{Opq(1)} \right) x_{p_o^{Opq(1)}, j, t_{p_o}^{Opq(1)}, t_j}^{Opq(1)} && \text{Schedule delay for passengers} \\
 & && \text{whose PDT earlier than the 1}^{\text{st}} \\
 & && \text{train's departure} \\
 & + \sum_{\substack{pq \in PQ \\ o \in Opq \mid o \neq 1, Opq(n) \\ (p_o^{Opq(o)}, j, u^{Opq(o)}, t_j) \in \psi^{Opq(o)}}} c_d^{pq} \times (w_{t_{p_o}^{Opq(o)}, L}^{Opq(o)} + w_{t_{p_o}^{Opq(o)}, R}^{Opq(o)}) \times x_{p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j}^{Opq(o)} && \text{Schedule delay for passengers} \\
 & && \text{whose PDT is between the 1}^{\text{st}} \\
 & && \text{and last train departures} \\
 & + \sum_{\substack{pq \in PQ \\ (p_o^{Opq(n)}, j, t_{p_o}^{Opq(n)}, t_j) \in \psi^{Opq(n)}}} c_d^{pq} \left( w_{t_{p_o}^{Opq(n)}, L}^{Opq(n)} + w_{t_{p_o}^{Opq(n)}, R}^{Opq(n)} \right) x_{p_o^{Opq(n)}, j, t_{p_o}^{Opq(n)}, t_j}^{Opq(n)} && \text{Schedule delay for passengers} \\
 & && \text{whose PDT later than the last} \\
 & && \text{train's departure} \\
 & + \sum_{\substack{(r, r') \in Z \\ (t, t') \in L_{r, r'}}} d^r (t' - t) y_{t, t'}^{r, r'} && \text{Penalty for trains staying} \\
 & && \text{longer than scheduled stop} \\
 & && \text{time at stations}
 \end{aligned}$$

# Constraints

- Unique departure from origin
- Unique sinking at the destination
- Flow conservation
- Linkage between trains
- Equal number of trains in each direction
- Blocks capacity constraint
- Block transition constraint
- Headway management
- Maintaining the order of passenger trains
- Binary variables

# Solution approach

- Because passenger schedule delay depends on the location of two neighboring trains, the above problem at the top (passenger) level is quadratic
- Directly solving QIP is difficult
- Reformulate the problem as a Linear Binary Program (LBP)
  - Each quadratic term in the objective function is replaced by a new variable
  - Three new constraints are added for each new variable to maintain the relationships among variables
  - Reduce the number of constraints using the special structure of the problem

# Numerical analysis

- A small problem
- Effect of speed heterogeneity
- A more realistic problem

# Numerical analysis

## A small problem

- Set up:
  - 11 blocks: 6 track segments and 5 sidings
  - 2 O-D pairs (one in each direction)
  - Each track segment 18 miles long
  - Sidings evenly distributed along the corridor, each 2 miles long
  - Total corridor length: 120 miles

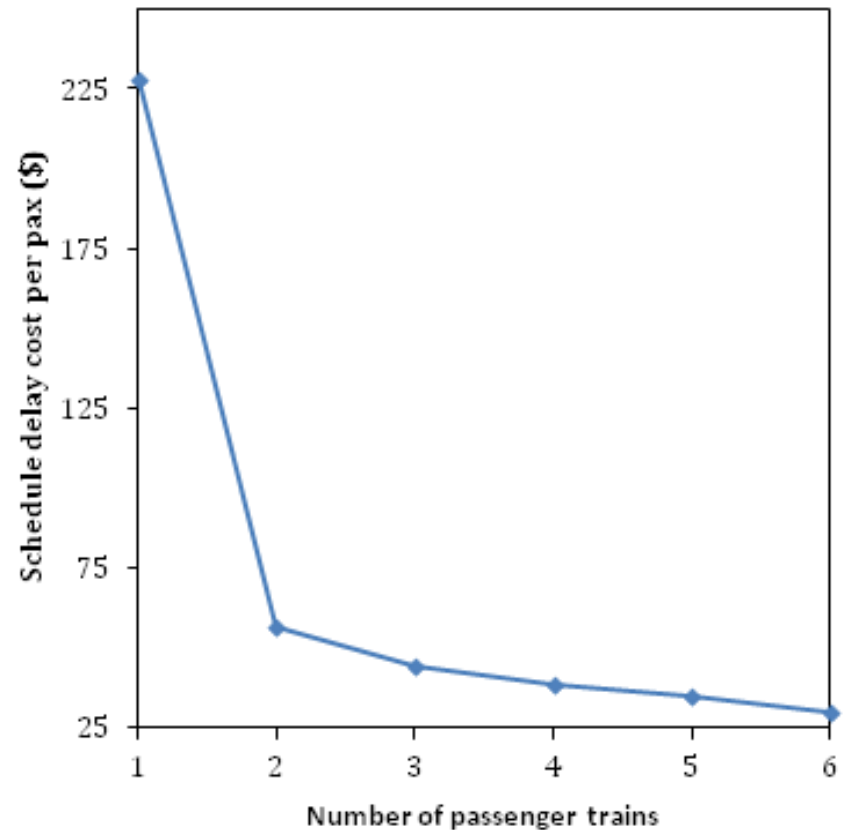
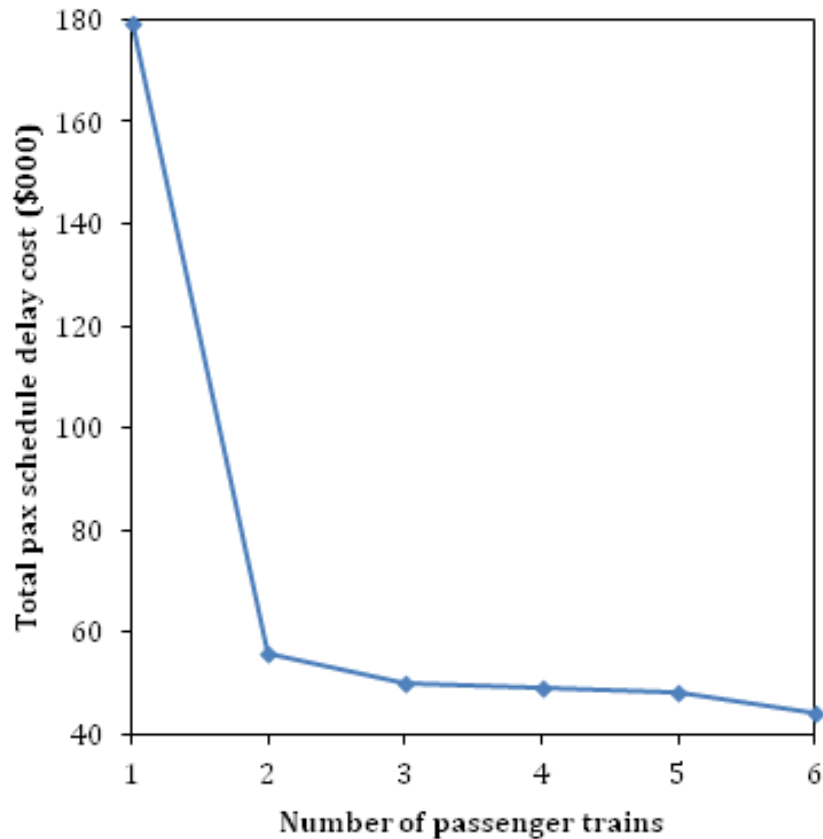
# Numerical analysis

## A small problem

- Set up (cont'd)
  - Operating speed
    - Freight trains: 60 mph
    - Passenger trains: 120 mph
  - Consider daily service frequency of 1-5 trains
  - Elastic passenger demand (elasticity: 0.4, based on Adler et al. (2010))

# A small problem: results

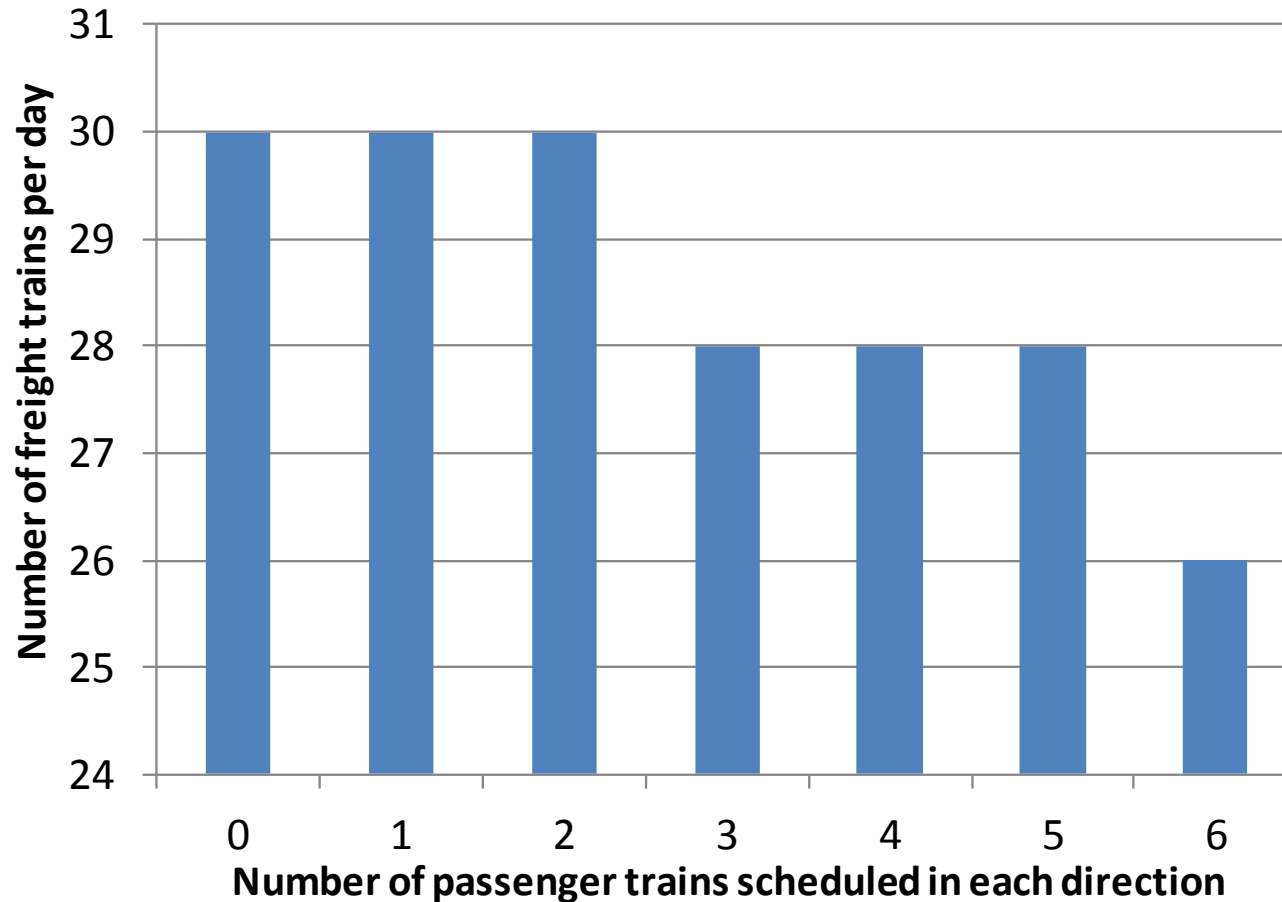
## passenger schedule delay cost





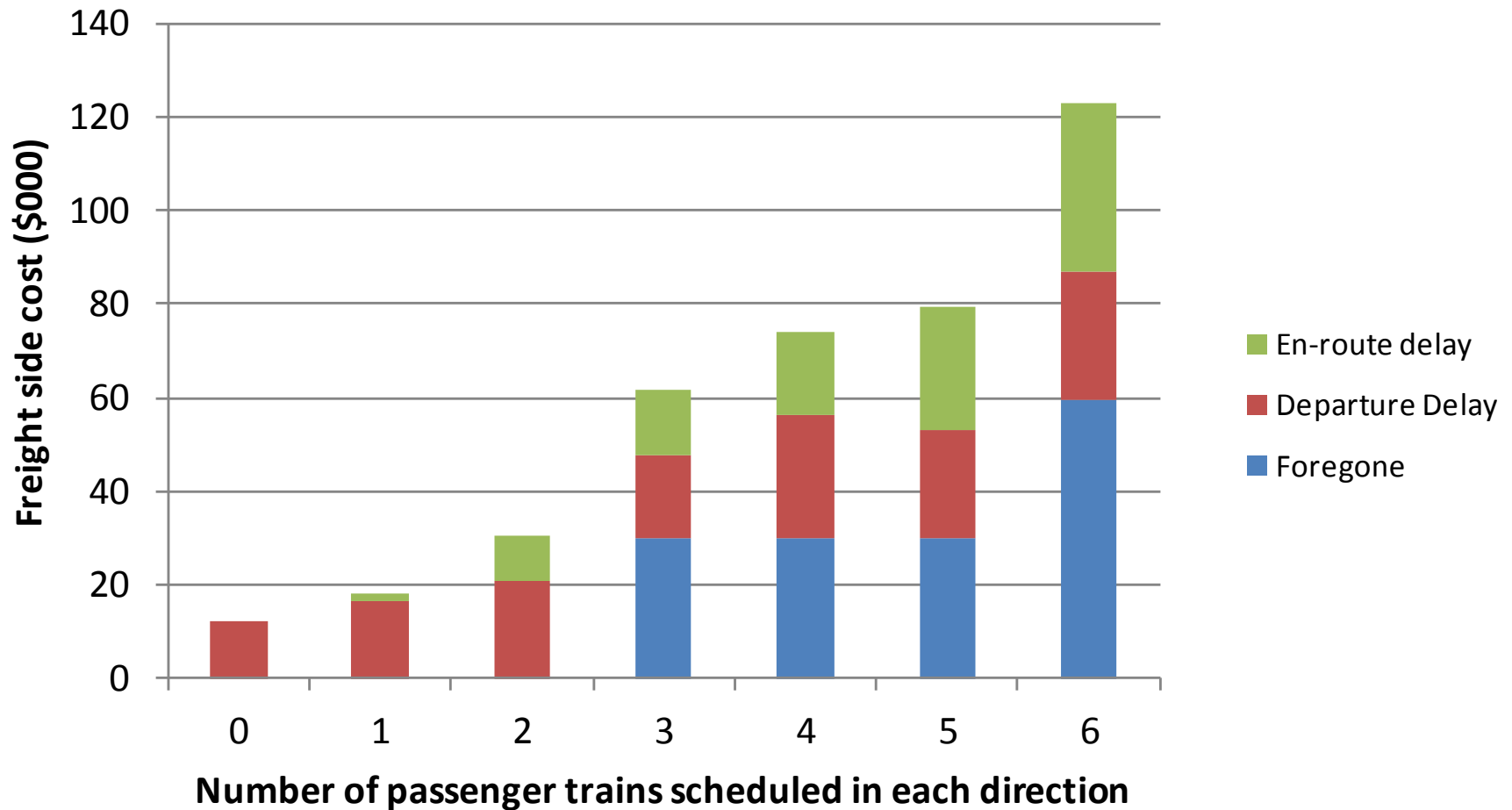
# A small problem: results

Freight trains that can be scheduled  
on the corridor



# A small problem: results

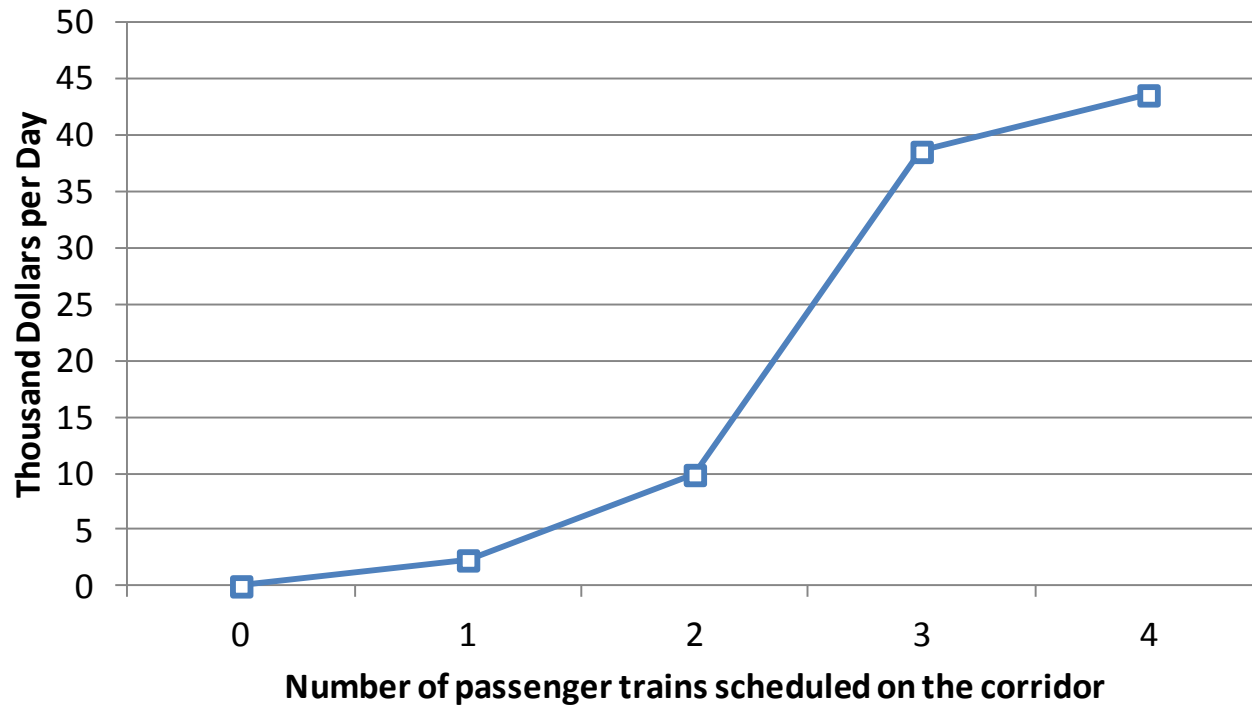
Freight side cost (\$000)



# A small problem: results

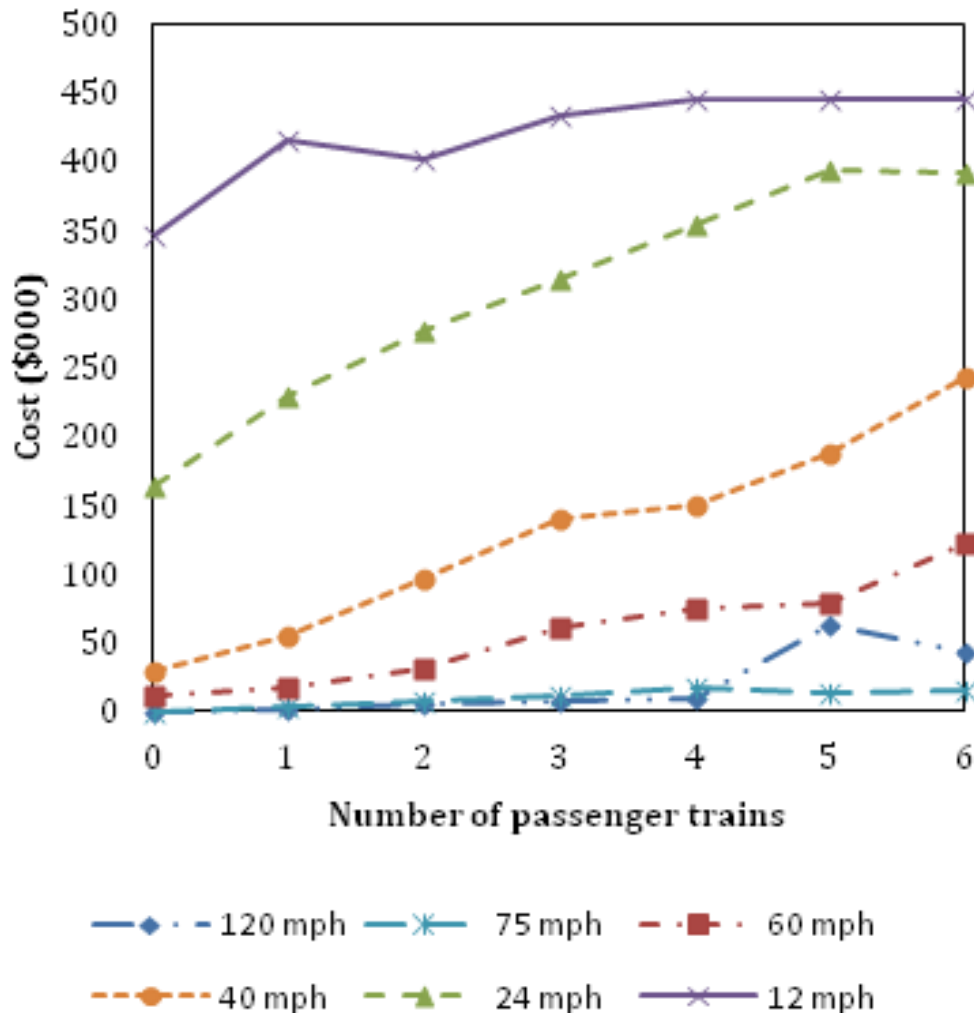
Impact on freight trains of giving scheduling priority to passenger trains

Price of giving scheduling priority to passenger trains



To investigate the impact of giving passenger trains scheduling priority, we optimize the sum of passenger and freight trains in “one shot”

# Impact of train speed heterogeneity on freight side cost



- Greater speed heterogeneity leads to higher freight side cost
- Sensitivities of freight side cost to number of passenger trains vary by speed

# A larger (and more realistic) problem (on-going efforts)

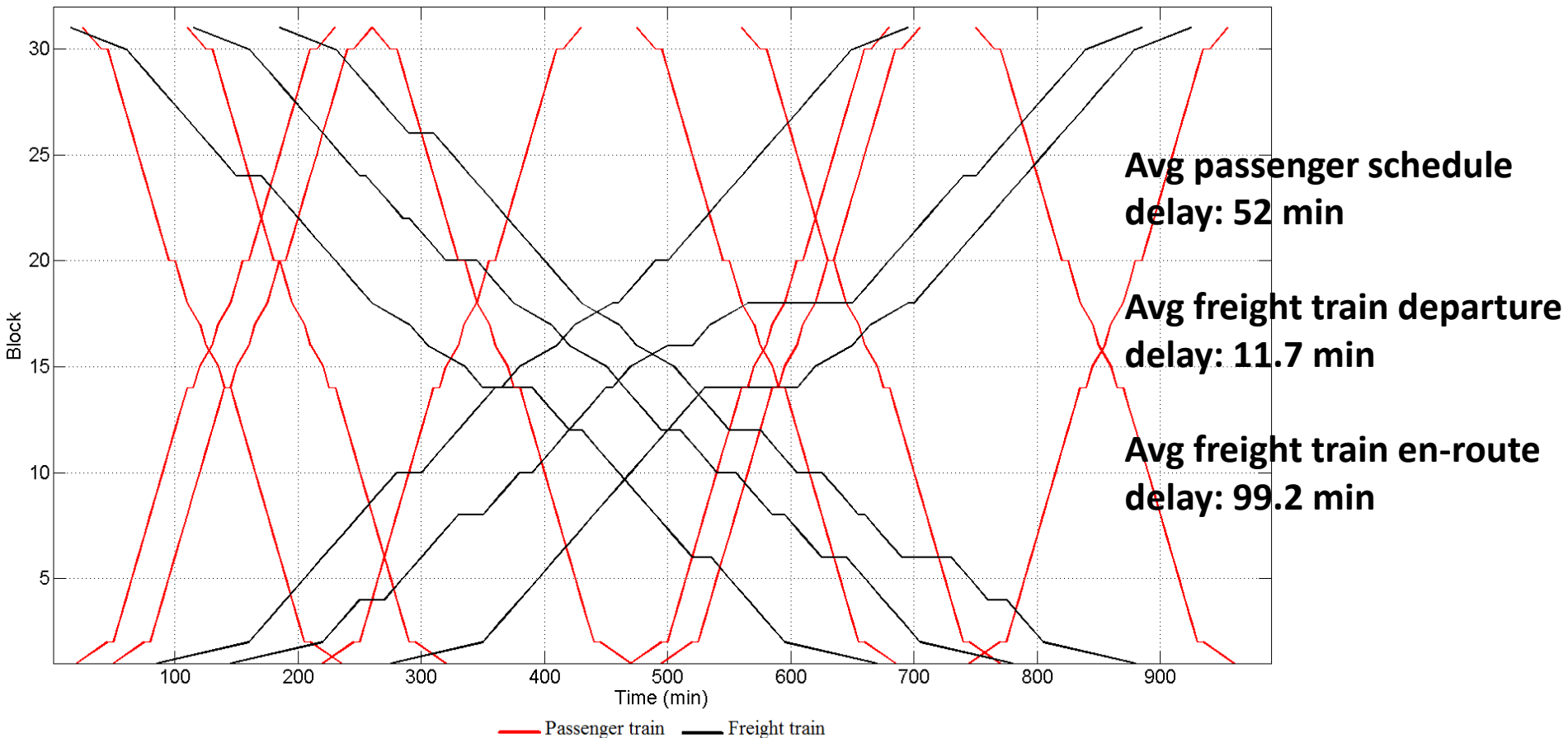
- Set up
  - Try to reflect to the maximum extent the future Chicago-St Louis HSR line
  - 285 mile-long shared use corridor
  - 18 double track and 15 single track segments
  - Consider 6 daily passenger trains running at an average speed of 90 mph (accounting for acceleration and deceleration)
  - Freight train average speed at 30 mph

# A larger (and more realistic) problem (on-going efforts)

- Consider two ends and four intermediate stations on the Chicago-St Louis Corridor
- O-Ds among these stations account for more than 95% of total O-D traffic

Station	Number of passengers per day	
	Southbound	Northbound
Chicago	1,767	0
Joliet	198	13
Normal	130	600
Springfield	154	436
Alton	9	253
St. Louis	0	944
Total	2,258	2,246

# A larger (and more realistic) problem (on-going efforts)



# Summary (I)

## Contributions to planning methodology

- Proposed a two-level modeling framework for shared use rail corridor planning
- Comprehensive consideration of cost and time components, including passenger schedule delay, foregone and elastic demand
- Employed a hypergraph based modeling approach which is more capable of dealing with train conflicts
- Designed an efficient solution approach to solve the planning problem within short computation time



# Summary (II)

## Policy findings and implications

- Schedule delay will be an important component in passenger generalized travel cost
- Marginal schedule delay cost reduction is diminishing with number of passenger trains
- Some freight trains will be forced out of service, and foregone demand cost will substantially increase as more passenger services are scheduled
- Costs to freight railroads from giving scheduling priority to passenger trains can be in the order of tens of thousands dollars per day

# Summary (III)

## Policy findings and implications

- Speed heterogeneity is important in affecting freight side cost: it may be desirable to increase freight train speed when HSR is introduced to shared corridors
- Overall, this study presents a beginning of systematic, quantitative analysis of shared use rail corridors. The interactions between passenger and freight rail deserve further attention and study.

**Thank you!**  
**Questions?**

# Back-up slides

# Order among passenger trains

- Two approaches are proposed to maintain the order among passenger trains
- First approach: we penalize combinations of starting arcs of two consecutive trains which violate order of trains with a very large number,  $M$ .

$$\sum_{o \in O^{pq} \mid o \neq 1} M \times x_{p_o^{O^{pq}(o-1)}, j, t_{p_o^{O^{pq}(o-1)}, t_j}}^{O^{pq}(o-1)}}^{O^{pq}(o-1)} \times x_{p_o^{O^{pq}(o)}, j, t_{p_o^{O^{pq}(o)}, t_j}}^{O^{pq}(o)}}^{O^{pq}(o)}$$

$$(t_{p_o^{O^{pq}(o-1)}, t_{p_o^{O^{pq}(o)}}}^{O^{pq}(o-1)}) \mid t_{p_o^{O^{pq}(o-1)} \geq t_{p_o^{O^{pq}(o)}}^{O^{pq}(o-1)}$$

$$(p_o^{O^{pq}(o-1)}, j, t_{p_o^{O^{pq}(o-1)}, t_j}) \in \psi^{O^{pq}(o-1)}$$

$$(p_o^{O^{pq}(o)}, j, t_{p_o^{O^{pq}(o)}, t_j}) \in \psi^{O^{pq}(o)}$$

# Order among passenger trains

- Second approach: introduce a new set of constraints, in which we ban simultaneous selection of variables violating the order of trains

$$\sum_{(p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j) \in \psi^{Opq(o-1)} | t_{p_o}^{Opq(o-1)} \geq t_{p_o}^{Opq(o)}} x_{p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j}^{Opq(o-1)} + x_{p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j}^{Opq(o)} \leq 1$$

$$\forall \{t_{p_o}^{Opq(o)} | (p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, v) \in \psi^{Opq(o)}\}, \{o \in Opq | o \neq 1\}, pq \in PQ$$

# Constraints (1)

- Linear network constraints
  - Unique departure from origin

$$\sum_{(p_o^r, j, t_i, t_j) \in \Psi^r} x_{p_o^r, j, t_i, t_j}^r \leq 1 \quad \forall \{r \in R | (r, r') \notin Z\}$$

- Flow conservation

$$\sum_{(a, i, t_i, t) \in \Psi^r} x_{a, i, t_i, t}^r = \sum_{(i, j, t, t_j) \in \Psi^r} x_{i, j, t, t_j}^r \quad \forall r \in R, \{i \in B | i \neq p_o^r\}, t \in T$$

- Unique sinking at the destination

$$\sum_{(p_d^r, e^r, t_i, t_j) \in \Psi^r} x_{p_d^r, e^r, t_i, t_j}^r \leq 1 \quad \forall \{r \in R | (r, r') \notin Z\}$$

# Constraints (2)

- Linear network constraints

- Linkage between trains

$$\sum_{(p_d^r, e^r, t_i, t) \in \Psi^r} x_{p_d^r, e^r, t_i, t}^r = \sum_{(t, t') \in L_{r, r'}} y_{t, t'}^{r, r'} \quad \forall (r, r') \in Z, t \in T$$

$$\sum_{(p_o^{r'}, j, t', t_j) \in \Psi^{r'}} x_{p_o^{r'}, j, t', t_j}^{r'} = \sum_{(t, t') \in L_{r, r'}} y_{t, t'}^{r, r'} \quad \forall (r, r') \in Z, t \in T$$

- Equal number of trains on each direction (optional)

$$\sum_{\substack{r \in R^N \mid (r, r') \notin Z \\ (p_d^r, e^r, t_i, t_j) \in \Psi^r}} x_{p_d^r, e^r, t_i, t_j}^r = \sum_{\substack{r \in R^S \mid (r', r) \notin Z \\ (p_o^{r'}, j, t_i, t_j) \in \Psi^{r'}}} x_{p_o^{r'}, j, t_i, t_j}^{r'}$$

- Integer variables

$$x, y \in \{0, 1\}$$



# Constraints (3)

- Side constraints:

- Blocks capacity constraint:

$$\sum_{\substack{r \in R \\ (i,j,u,v) \in \Psi^r | u \leq t < v}} x_{i,j,u,v}^r \leq b_t^i \quad \forall i \in B, t \in T$$

- Block transition constraint:

$$\sum_{\substack{r \in R^N \\ v \in \{t+1-\varepsilon, \dots, t+1+\delta\} \\ (i,j,u,v) \in \Psi^r | j=a+1, j \neq i}} x_{i,j,u,v}^r + \sum_{\substack{r \in R^S \\ v \in \{t+1-\varepsilon, \dots, t+1+\delta\} \\ (i,j,u,v) \in \Psi^r | j=a, j \neq i}} x_{i,j,u,v}^r \leq v_t^a \quad \forall (a, t) \in \mathcal{T}$$

- Headway management:

$$\sum_{\substack{r \in R^N | h^r \geq 1 \\ a \in \{i-h, \dots, i-1\} \\ (a,j,t_i,t_j) \in \Psi^r | t_i \leq t < t_j, a \neq j}} x_{a,j,t_i,t_j}^r + \sum_{\substack{r \in R^N \\ (i,j,t_i,t_j) \in \Psi^r | t_i \leq t < t_j}} x_{i,j,t_i,t_j}^r \leq b_t^i \quad \forall i \in B, t \in T$$

$$\sum_{\substack{r \in R^S | h^r \geq 1 \\ a \in \{i+1, \dots, i+h\} \\ (a,j,t_i,t_j) \in \Psi^r | t_i \leq t < t_j, a \neq j}} x_{a,j,t_i,t_j}^r + \sum_{\substack{r \in R^S \\ (i,j,t_i,t_j) \in \Psi^r | t_i \leq t < t_j}} x_{i,j,t_i,t_j}^r \leq b_t^i \quad \forall i \in B, t \in T$$

# Objective function

## Part 2: Passenger waiting time cost

- All passengers arriving before the departure time of the first train take this train.
- Passengers who preferred to depart between departure of the first train and the middle of the departures of the first and the second train also tend to take the first train
- Passengers arriving between  $(t_{p_o}^{O^{pq}(n-1)} + t_{p_o}^{O^{pq}(n)})/2$  and  $t_{p_o}^{O^{pq}(n)}$  take the last train
- All passengers arriving after the departure time of the last train take this train
- Passengers with DDT between  $(t_{p_o}^{O^{pq}(o)} + t_{p_o}^{O^{pq}(o-1)})/2$  and  $(t_{p_o}^{O^{pq}(o)} + t_{p_o}^{O^{pq}(o+1)})/2$  take train  $O^{od}(o)$

# Overall optimization problem

- Decision variable vector  $x$  is composed of  $x_{i,j,t_i,t_j}^r$   
 $Max f \cdot x + \frac{1}{2} x^T H x$   
*Subject to*  
 $Ax \leq b$   
 $x = 0 \text{ or } 1$
- Binary variable denoting the occupancy by train  $r$  of node  $i$  at time  $u$  which exits into node  $j$  at time  $v$

# Reformulating the problem

$$Z_{t_{p_o}^{Opq(o-1)} t_{p_o}^{Opq(o)}} = x_{p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j}^{Opq(o-1)} \cdot x_{p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j}^{Opq(o)}$$

$$\forall \{o \in Opq | o \neq 1\}, \{pq \in PQ\}, \left\{ \left( t_{p_o}^{Opq(o-1)}, t_{p_o}^{Opq(o)} \right) \mid t_{p_o}^{Opq(o-1)} < t_{p_o}^{Opq(o)} \right\},$$

$$\left\{ \left( p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j \right) \in \psi^{Opq(o-1)} \right\}, \left\{ \left( p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j \right) \in \psi^{Opq(o)} \right\}$$

$$Z_{t_{p_o}^{Opq(o-1)} t_{p_o}^{Opq(o)}} \leq x_{p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j}^{Opq(o-1)}$$

$$Z_{t_{p_o}^{Opq(o-1)} t_{p_o}^{Opq(o)}} \leq x_{p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j}^{Opq(o)}$$

$$x_{p_o^{Opq(o-1)}, j, t_{p_o}^{Opq(o-1)}, t_j}^{Opq(o-1)} + x_{p_o^{Opq(o)}, j, t_{p_o}^{Opq(o)}, t_j}^{Opq(o)} \leq 1 + Z_{t_{p_o}^{Opq(o-1)} t_{p_o}^{Opq(o)}}$$

# Ongoing efforts

- Analyzing the effects of speed heterogeneity on optimal schedules
- Sensitivity analysis to parameter values
- Developing heuristics to tackle problems with bigger size
- Capacity investment decision making